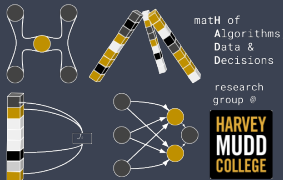


# An Interpretable Joint Nonnegative Matrix Factorization-Based Point Cloud Distance Measure

**by** Jamie Haddock  
(Harvey Mudd College, Department of Mathematics)  
**on** March 23, 2023,  
Conference on Information Sciences and Systems (CISS)

joint with Hannah Friedman, Amani R. Maina-Kilaas, Julianna Schalkwyk, and  
Hina Ahmed (graduating Harvey Mudd College and Pitzer College seniors)

supported by NSF DMS #2211318



# Motivation

# » Dataset similarity

... my migraines. Of course I have heart issues too, but the migraines are my main concern right now. My priority is getting

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... My doctor was great, realized it was a heart attack really quick. I didn't quite know what

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... chest pain. I had been feeling lightheaded and nauseous. The pain was definitely there but really I felt more a tightness in my chest than anything. It left me short of breath, which was probably making me lightheaded. The EKG indicated that my heart had several blockages that would need a stent. My cardiologists were able to clear the blockages and I spent one night under watch in the hospital.

After my heart attack, I completely changed my lifestyle. I quit smoking, started an exercise regimen and diet...

... I recently had a minor procedure where I was under anesthesia for it. Whenever I woke up, I had pain in my jaw (which the

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... I woke up with a slightly sore throat, by 12 p.m. I started work at a buddies house, I've been

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... had a lipoma (a bit over an inch around) over my right shoulder blade for years now. Never hurt at all before, until 3 days ago it started

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... roughly a year ago I was sitting in the office drinking an energy drink when I started to get this bad tingling sensation in my neck which caused great discomfort. Figuring out the energy drink was causing this I cut it out of my "diet". With that the pain and problems went away. But slowly (over the course of months) one by one different foods and drink have now that same effect mostly being sugars/alcohol/caffeine. The pain I get is very isolated at the left and right occiput. Depending on what I ingest the pain I get might flow down to lower in my neck...

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Understanding the similarities and differences between datasets arises in many contexts: e.g., transfer learning, plagiarism/manipulation detection, and data denoising.

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Patient Surveys

Patients

heart	3	3	0	1
weakness	2	0	0	0
chest	0	2	0	0
migraine	0	0	2	3
lightheaded	0	2	2	1
pain	3	2	2	4
.				
.				
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Term-Document Matrix

## » Point Cloud Distances

**Chamfer's distance:**

$$d_{\text{cham}}(X_1, X_2) = \frac{1}{|X_1|} \sum_{\mathbf{x} \in X_1} \min_{\mathbf{y} \in X_2} \|\mathbf{y} - \mathbf{x}\|_2^2 + \frac{1}{|X_2|} \sum_{\mathbf{y} \in X_2} \min_{\mathbf{x} \in X_1} \|\mathbf{x} - \mathbf{y}\|_2^2$$

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We seek a distance that is:

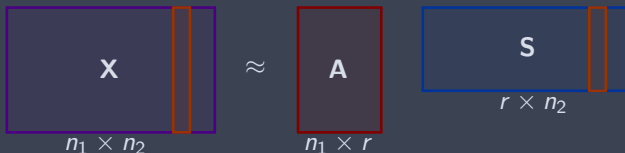
- \* More robust to outliers.
- \* Utilizes the structure of data.
- \* Helps illustrate how the data is similar or dissimilar.

# Introduction

## » Nonnegative Matrix Factorization (NMF)

**Model:** Given nonnegative data  $\mathbf{X}$ , compute nonnegative  $\mathbf{A}$  and  $\mathbf{S}$  of lower rank so that

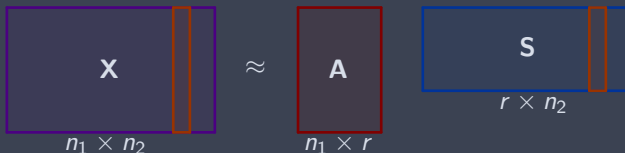
$$\mathbf{X} \approx \mathbf{AS}.$$



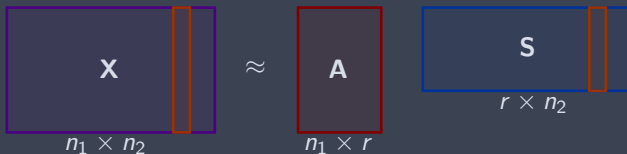
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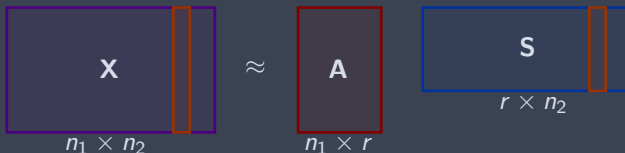


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**Motivation**

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**Introduction**

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**Our Method and Distance**

○○○

**Experiments**

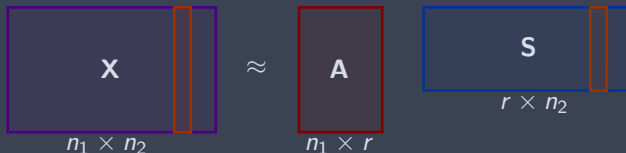
○○○○○

**Conclusions**

○○○



## » Nonnegative Matrix Factorization (NMF)



- ▷ Popularized by [Lee & Seung 1999]
- ▷ Employed for dimensionality-reduction and topic modeling
- ▷ Often formulated as

$$\min_{\mathbf{A} \in \mathbb{R}_{\geq 0}^{n_1 \times r}, \mathbf{S} \in \mathbb{R}_{\geq 0}^{r \times n_2}} \|\mathbf{X} - \mathbf{AS}\|_F^2 \quad \text{or} \quad \min_{\mathbf{A} \in \mathbb{R}_{\geq 0}^{n_1 \times r}, \mathbf{S} \in \mathbb{R}_{\geq 0}^{r \times n_2}} D(\mathbf{X} \| \mathbf{AS}).^1$$

---

<sup>1</sup>information divergence  $D(\mathbf{A} \| \mathbf{B}) = \sum_{i,j} \left( \mathbf{A}_{ij} \log \frac{\mathbf{A}_{ij}}{\mathbf{B}_{ij}} - \mathbf{A}_{ij} + \mathbf{B}_{ij} \right)$

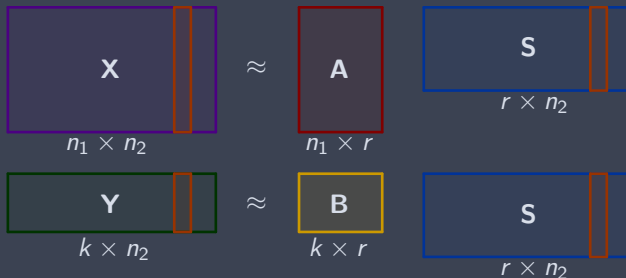
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**Example:** Semi-supervised NMF



Often applied in classification!

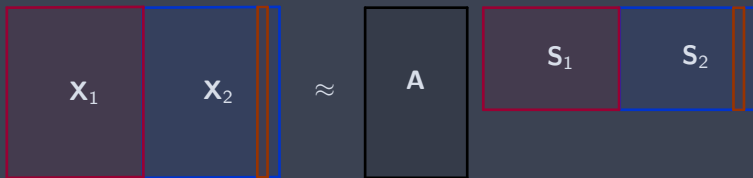
<sup>1</sup>Lee, H., Yoo, J., and Choi, S. "Semi-supervised nonnegative matrix factorization." IEEE Signal Processing Letters 17.1 (2009): 4-7.

H., et al. "Semi-supervised Nonnegative Matrix Factorization for Document Classification." 2021 55th Asilomar Conference on Signals, Systems, and Computers. IEEE, 2021.

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**Model:** Jointly factorize two nonnegative matrices  $\mathbf{X}_1$  and  $\mathbf{X}_2$ , sharing one factor matrix between the factorizations.

**Example:** Joint NMF/Guided NMF



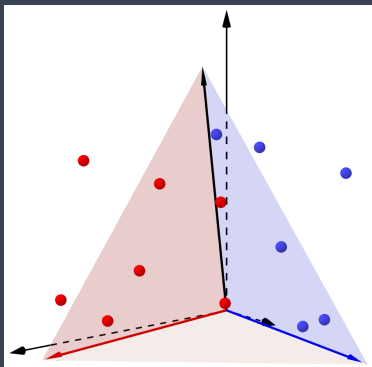
**Intuition:** many columns of  $A$  used in representing  $X_1$  and  $X_2$  indicates dataset similarity.

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<sup>1</sup>Kim, H., et al. "Simultaneous discovery of common and discriminative topics via joint nonnegative matrix factorization." Proc. ACM SIGKDD Int. Conf. Knowl. Disc. Data Mining. 2015.

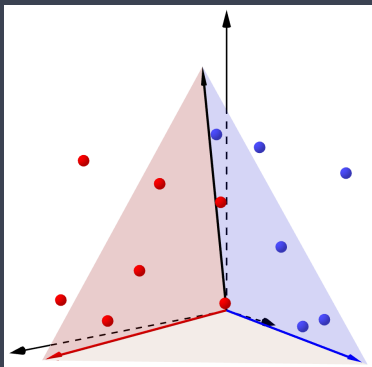
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## » Joint NMF (jNMF) for Similarity

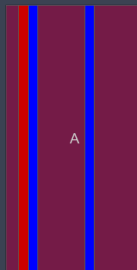


NMF learns a conic  
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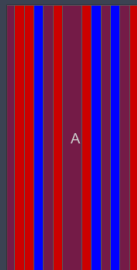
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Similar



Different

## Our Method and Distance

## » Our jNMF Similarity Method

Intuition: use the entries of  $S_1$  and  $S_2$  to measure how much topics are shared between datasets.

**Method:**



## » Our jNMF Similarity Method

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### Method:

- \* Scale each column in  $X_1, X_2$  to be mean one.
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$$s_i = \max \left( \{s_{ij}^{(1)}\}_{j=1}^{n_1} \cup \{s_{ij}^{(2)}\}_{j=1}^{n_2} \right)$$

where  $s_{i1}^{(1)}, s_{i2}^{(1)}, \dots, s_{in_1}^{(1)}$  and  $s_{i1}^{(2)}, s_{i2}^{(2)}, \dots, s_{in_2}^{(2)}$  are the entries of the  $i$ th rows of  $S_1$  and  $S_2$ , respectively.

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  - \* Compute  $\mathbf{p}_i^{(j)} := F_i^{(2)}(T_i) - F_i^{(1)}(T_i)$ , where

$$F_i^{(1)}(T_i) := \frac{1}{n_1} \sum_{j=1}^{n_1} \mathbf{1}[s_{ij}^{(1)} < T_i] \text{ and } F_i^{(2)}(T_i) := \frac{1}{n_2} \sum_{j=1}^{n_2} \mathbf{1}[s_{ij}^{(2)} < T_i].$$

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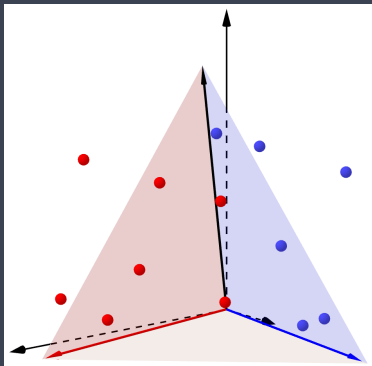
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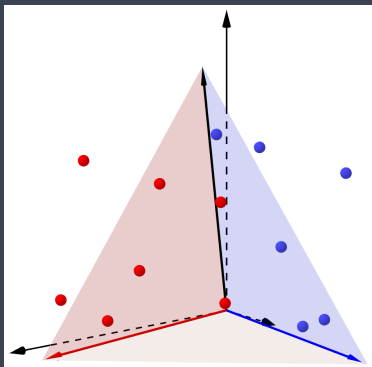
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- \* Return  $\bar{\mathbf{p}} = \frac{1}{K} \sum_{j=1}^K \mathbf{p}^{(j)}$

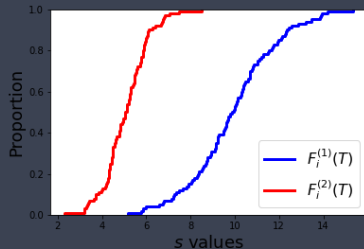
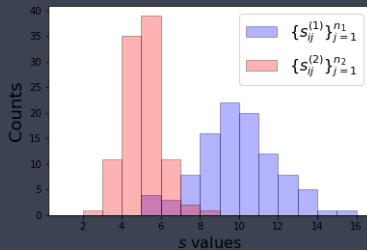
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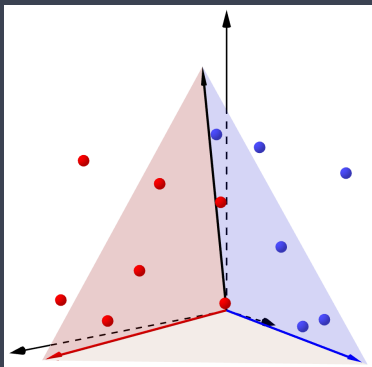


$$p_i^{(j)} := F_i^{(2)}(T_i) - F_i^{(1)}(T_i)$$



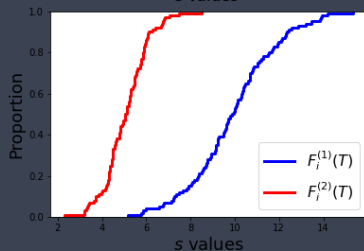
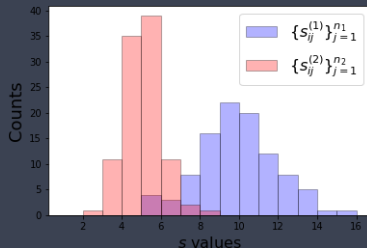


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$$d(X_1, X_2) := \|\bar{\mathbf{p}}\|_1$$

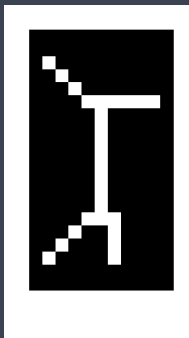
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# Experiments

# » Swimmer Image Dataset

## Swimmer Images

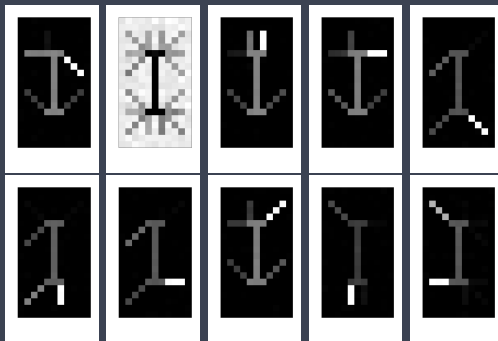
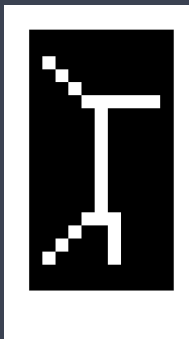


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<sup>1</sup>Donoho, D., and Stodden, V. "When does non-negative matrix factorization give a correct decomposition into parts?." NeurlPS (2003).

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Swimmer Images



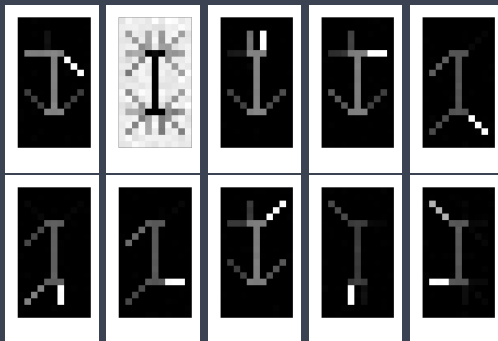
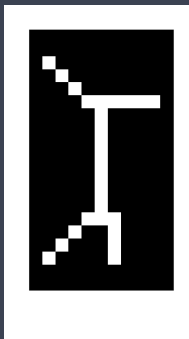
Basis vectors learned by jNMF on Swimmer dataset  $X_1, X_1 + N$  where  $N$  is uniform noise.

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Swimmer Images



Basis vectors learned by jNMF on Swimmer dataset  $X_1, X_1 + N$  where  $N$  is uniform noise.

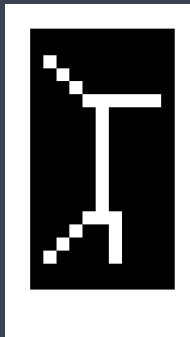
$$\bar{\mathbf{p}} = [0.063, -0.901, 0.076, 0.065, 0.069, \\ 0.058, 0.058, 0.069, 0.079, 0.079]$$

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<sup>1</sup>Donoho, D., and Stodden, V. "When does non-negative matrix factorization give a correct decomposition into parts?." NeurlPS (2003).

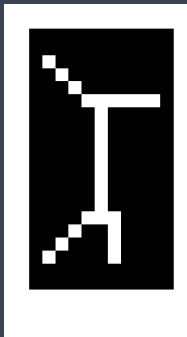
# » Swimmer Image Dataset

## Swimmer Images



# » Swimmer Image Dataset

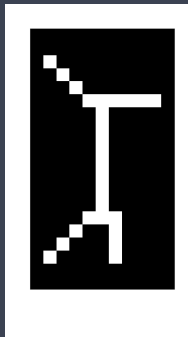
## Swimmer Images



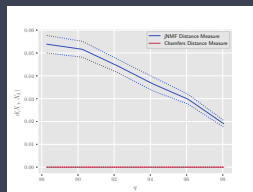
$X_2$	$X_1$	$X_1 P_\pi$	$\lambda X_1$	$\tilde{X}_1$	$X_1 + N$	$N$
$d(X_1, X_2)$	0.000	0.000	0.000	0.052	1.509	2.297
$d_{\text{cham}}(X_1, X_2)$	0.000	0.000	0.000	0.000	0.741	1.560

# » Swimmer Image Dataset

## Swimmer Images



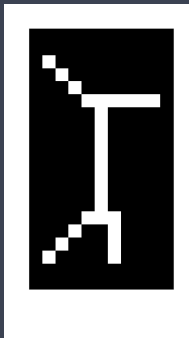
$X_2$	$X_1$	$X_1 P_\pi$	$\lambda X_1$	$\tilde{X}_1$	$X_1 + N$	$N$
$d(X_1, X_2)$	0.000	0.000	0.000	0.052	1.509	2.297
$d_{\text{cham}}(X_1, X_2)$	0.000	0.000	0.000	0.000	0.741	1.560



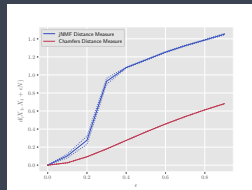
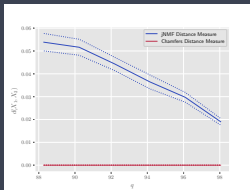


## » Swimmer Image Dataset

## Swimmer Images



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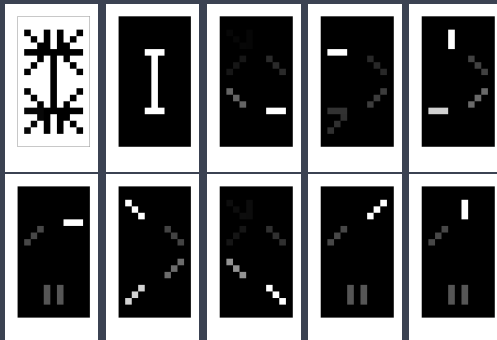
# » Swimmer Image Dataset

Swimmer and  
Inverse Swimmer:



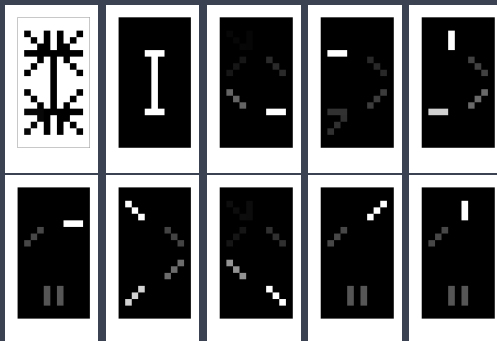
# » Swimmer Image Dataset

Swimmer and  
Inverse Swimmer:



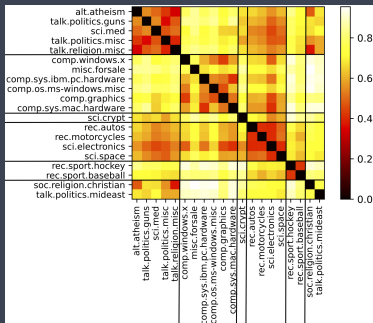
# » Swimmer Image Dataset

Swimmer and  
Inverse Swimmer:



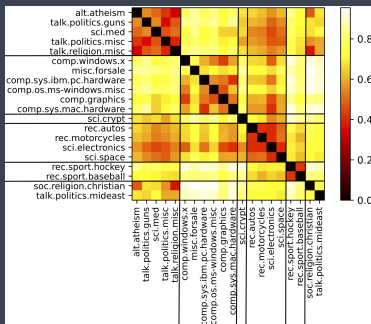
$$\bar{\mathbf{p}} = [-0.999, 1.000, 0.010, -0.017, 0.003, -0.004, 0.015, 0.004, -0.001, -0.000]$$

# » 20 Newsgroups Dataset

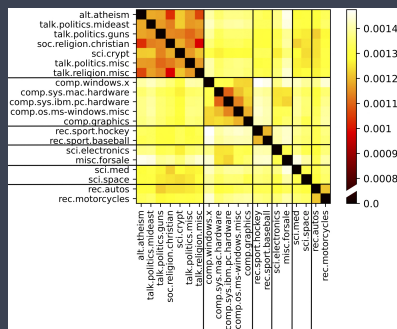


jNMF Distance

# » 20 Newsgroups Dataset



jNMF Distance

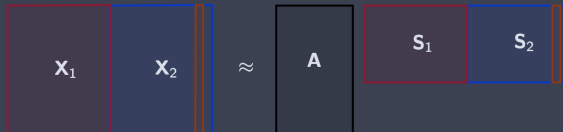


Chamfer Distance

# Conclusions

## » Conclusions

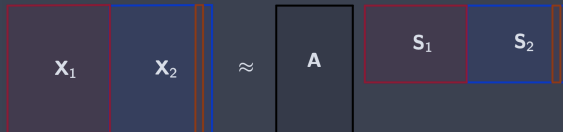
- ▷ jNMF provides information about dataset similarity and dissimilarity





## » Conclusions

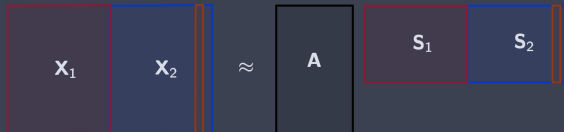
- ▷ jNMF provides information about dataset similarity and dissimilarity



- ▷ we can aggregate this information using samples from the empirical distribution function to form a vector indicating which dataset learned basis vectors represent,  $\bar{p}$

## » Conclusions

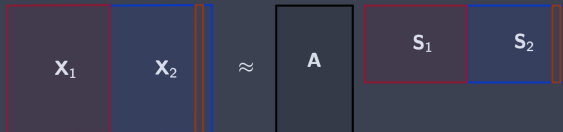
- ▷ jNMF provides information about dataset similarity and dissimilarity



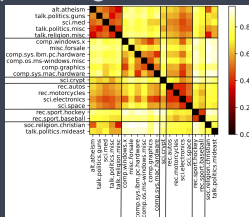
- ▷ we can aggregate this information using samples from the empirical distribution function to form a vector indicating which dataset learned basis vectors represent,  $\bar{p}$
- ▷ the information can be further aggregated to yield a distance,  $d(X_1, X_2) = \|\bar{p}\|_1$

## » Conclusions

- ▷ jNMF provides information about dataset similarity and dissimilarity



- ▷ we can aggregate this information using samples from the empirical distribution function to form a vector indicating which dataset learned basis vectors represent,  $\bar{p}$
- ▷ the information can be further aggregated to yield a distance,  $d(X_1, X_2) = \|\bar{p}\|_1$
- ▷ initial experiments are promising



» **Thanks for listening!**

## Questions?

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- [2] Hyekyoung Lee, Jiho Yoo, and Seungjin Choi. Semi-supervised nonnegative matrix factorization. IEEE Signal Processing Letters, 17(1):4–7, 2009.
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